6. $x+2y=100$

$x=100-2y$

$x=100-2(25)$

$x=50$

the product is a maximum when $x=50$ and $y=25$.

$P=xy$

$P=(100-2y)y$

$=100y-2y^2$

$P'=100-4y$

$100-4y=0$

$-4y=-100$

$y=25$

$P''=-4$

$P''(25)=-4$

$<0, \therefore$ a maximum

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8. $f(x)=\frac{1}{1+x^2}$ on $[-2,1]$

find the maximum and minimum value

$=(1+x^2)^{-1}$

$f'(x)=-(1+x^2)^{-2}(2x)$

$=\frac{-2x}{(1+x^2)^2}$

$\cancel{1+x^2=0}$ $\quad -2x=0$

$\cancel{x^2=-1}$ $\quad x=0$

$f(-2)=\frac{1}{1+4}$

$=1/5$

$f(0)=1$ $\quad f(1)=1/2$

maximum value at $(0,1)$

minimum at $(-2,1/5)$

Find two numbers whose product is -18 and the sum of whose squares is a minimum.

$$xy = -18 \qquad s = x^2 + y^2$$

$$y = -18/x \qquad s = x^2 + \left(-18/x\right)^2$$

$$y = \frac{-18}{3\sqrt{2}} \qquad = x^2 + \frac{324}{x^2}$$

$$= \frac{-6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \qquad = x^2 + 324x^{-2}$$

$$s' = 2x - 648x^{-3}$$

$$= \frac{-6\sqrt{2}}{2} \qquad = 2x - \frac{648}{x^3}$$

$$= -3\sqrt{2} \qquad = \frac{2x^4 - 648}{x^3}$$

the two numbers whose squares are a minimum are $x = 3\sqrt{2}$ and $y = -3\sqrt{2}$.

$$2x^4 - 648 = 0$$

$$2x^4 = 648$$

$$x^4 = 324$$

$$x = \pm 3\sqrt{2}$$

$$s'' = 2 + 1944x^{-4}$$

$$= 2 + \frac{1944}{x^4}$$

$$s''(3\sqrt{2}) = 2 + \frac{1944}{(3\sqrt{2})^4}$$

> 0, $\therefore$ a minimum

$y = 3x^4 - 6x^2 + 5/3$

$y' = 12x^3 - 12x$

$12x^3 - 12x = 0$

$12x(x^2 - 1) = 0$

$x = 0 \qquad x^2 - 1 = 0$

C.N. $\qquad x^2 = 1$

$x = \pm 1$

T.P. $\quad -2 \quad -\tfrac{1}{2} \quad \tfrac{1}{2} \quad 2$

C.N. $\quad - \quad -1 \quad + \quad 0 \quad - \quad 1 \quad +$

$y'(-2) = 12(-8) + 24 \qquad y'(\tfrac{1}{2}) = 12(\tfrac{1}{8}) - 6$

$y'(\tfrac{-1}{2}) = 12(\tfrac{-1}{8}) + 6 \qquad y'(2) = 12(8) - 24$

decreasing on $(-\infty, -1)$ and $(0, 1)$

increasing on $(-1, 0)$ and $(1, \infty)$

$y'' = 36x^2 - 12$

$= 12(3x^2 - 1)$

T.P. $\quad -1 \quad 0 \quad 1$

I.P. $\quad + \quad -.577 \quad - \quad .577 \quad +$

$3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = 1/3$

$x = \pm \frac{1}{\sqrt{3}}$

$x \approx \pm .577$

$f''(-1) = 36 - 12$

$f''(0) = -12$

$f''(1) = 36 - 12$

Concave up on $(-\infty, -1/\sqrt{3})$ and $(\frac{1}{\sqrt{3}}, \infty)$

Concave down on $(-1/\sqrt{3}, 1/\sqrt{3})$

find relative extrema

$y = x^3 - 27x + 3$

$y' = 3x^2 - 27$

$3x^2 - 27 = 0$

$3x^2 = 27$

$x^2 = 9$

$x = \pm 3$

$y'' = 6x$

$y''(-3) = -18$ $y''(3) = 18$

< 0, ∴ a maximum > 0, ∴ a minimum

$y(-3) = -27 + 81 + 3$

$= 57$

relative maximum at $(-3, 57)$

$y(3) = 27 - 81 + 3$

$= -51$

relative minimum at $(3, -51)$

$f(x) = \frac{1}{2}x - \cos x$ on $(0, 2\pi)$

find relative extrema

$f'(x) = \frac{1}{2} + \sin x$

$\frac{1}{2} + \sin x = 0$

$\sin x = -\frac{1}{2}$

$x = \arcsin\left(-\frac{1}{2}\right)$

$x = 7\pi/6 \qquad x = \frac{11\pi}{6}$

$f''(x) = \cos x$

$f''(7\pi/6) = \cos 7\pi/6 = -\sqrt{3}/2$

$f''(11\pi/6) = \cos 11\pi/6 = \sqrt{3}/2$

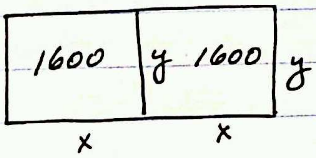
relative maximum at $\left(7\pi/6, \frac{7\pi + 6\sqrt{3}}{12}\right)$

relative minimum at $\left(11\pi/6, \frac{11\pi - 6\sqrt{3}}{12}\right)$

$$f(7\pi/6) = \frac{1}{2}(7\pi/6) - \cos 7\pi/6 = \frac{7\pi}{12} + \frac{\sqrt{3}}{2} = \frac{7\pi + 6\sqrt{3}}{12}$$

$$f\left(\frac{11\pi}{6}\right) = \frac{1}{2}\left(\frac{11\pi}{6}\right) - \cos 11\pi/6 = \frac{11\pi}{12} - \frac{\sqrt{3}}{2} = \frac{11\pi - 6\sqrt{3}}{12}$$

A farmer wishes to fence two identical adjacent, rectangular pens each with 1600 square feet of area. What are x and y so that the least amount of fence are required?



$$xy = 1600$$

$$y = \frac{1600}{x}$$

$$y = \frac{1600}{20\sqrt{3}}$$

$$= \frac{80}{\sqrt{3}}$$

$$= \frac{80\sqrt{3}}{3}$$

least amount of fence is when $x = 20\sqrt{3}$ and $y = 80\sqrt{3}/3$.

$$P = 4x + 3y$$

$$P = 4x + 4800x^{-1}$$

$$P' = 4 - 4800x^{-2}$$

$$= 4 - 4800/x^2$$

$$= \frac{4x^2 - 4800}{x^2}$$

$$4x^2 - 4800$$

$$4x^2 = 4800$$

$$x^2 = 1200$$

$$x = 20\sqrt{3}$$

$$P'' = 9600x^{-3}$$

$$= 9600/x^3$$

$$P''(20\sqrt{3}) = \frac{9600}{(20\sqrt{3})^3}$$

>0, so a minimum