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AP Calculus

Class Environment

Our class meets for approximately 55 minutes each day, and students should expect to have 30 to 45 minutes of homework assigned in each of their academic classes.

Course Sequence Leading to Calculus

Students take Honors Algebra 2, Honors Geometry, Honors Algebra 3, and Honors Precalculus as prerequisites for AP Calculus. Students who have not been on the honors track will additionally be required to take Algebra 3 prior to the Precalculus course.

Student Selection

Each year, seniors are advised by the department as to which mathematics course would be most likely to serve them best. If a student wishes to take a course above the level of the one recommended, he must meet with the department chair and his current teacher to request a different course. Almost always, students take the advice offered after such a conference. Some students select a course that is not an AP Calculus course even though they have been nominated. Such students are usually either carrying a heavy AP course load or have committed to significant leadership roles for the coming year and recognize that mathematics is not their highest priority.

AB Course Outline

The following is an outline of the topics we cover and a typical sequence in which those topics are covered. The time spent is only an estimate of the average number of days allotted to the topic because the actual time varies from year to year depending upon the students' abilities and interest, and also upon the richness of the class discussion that is generated. Also, with the wealth of interesting problems that are being supplied by those committed to reform calculus, and with the always changing capabilities of technology, it is difficult to anticipate extra days a class might spend in exploration or discovery. Each my students will be issued a graphing calculator for his/her use, and is expected to have it in class each day. The calculator of choice for our mathematics department is the TI-83Plus, however students use the TI-89 in AP Calculus. In my class, we use graphing calculators daily to explore, discover, and reinforce the concepts of calculus. Students may use the graphing calculators on some, but not all, assessments.

Resource materials used by the instructor for the class are the textbook Calculus of a Single Variable by Larson, Hostetler, and Edwards, Multiple-choice & Free-response Questions in Preparation for the AP Calculus (AB) Examination by David Lederman, and 2006 – 2007 Professional Development Workshop Materials from the College Board.

Local Linearity (This replaces the early review chapter in most traditional texts.)

Time: Approximately 8 days
In this unit, we review graphs of basic families of functions, linear equations, and algebraic simplification, all in a context which is new to the student and which leads him to the formal definition of the derivative.

1. Using the zoom and trace features of graphing calculators, we discover that almost all curves "straighten out" around a point (a, *f* (a)) when viewed close in. We write linear equations that seem to approximate the zoomed-in graph and then zoom-out to discover a (nearly) tangent line. This is done with a variety of functions: polynomial, exponential, logarithmic, trigonometric, and piece-wise defined.
2. Using algebraic functions, a particular point $P(x_o,f(x_o))$ is selected, and then the slope of a secant chord joining P with R(x, f(x)) is simplified. The class investigates this slope value for various x-values, discovering what happens when x is very close in value to x_o.
3. Using these same functions, the selected point becomes P(a, f(a)) and the general secant slope is discovered. The class then attempts to determine an expression that is likely to predict the secant slope value if x and a are extremely close in value.

Functions and Limits

Time: Approximately 15 days

Note 1: I do MUCH less formal work with limits than I used to as this is covered extensively in the Precalculus classes, but I do want my students to understand why some limits exist and why others do not. We no longer do limit proofs, but we do spend time discussing why "closeness" is a relative term, and then look at the limit definitions to see how a mathematician resolves the issue. I also emphasize that with regard to what happens at x = a in not relevant.

Note 2: The homework assignments given in this section also include problems that require the student to continue the investigation of the introductory work begun in our first unit. As limit notation is introduced, we incorporate that notation into our study of the limiting value of the secant segment's slope as x approaches a.

1. Review function notation, domain, and range.
2. Odd and even function definitions and graphical properties.
3. Introduction of limits intuitively.
4. Limit notation, including right and left-hand limits.
5. Understanding and describing asymptotic behavior of rational and exponential functions using limit notation.
6. Understanding asymptotes in terms of general graphical behavior and in terms of limits involving infinity.
7. Estimating limits from graphs, and from tables of values.
8. Calculating limits using algebra.

9. Analysis of graphs using technology such as the TI-89 calculator and Winplot software to predict and explain local and global behavior of a function.
10. Presentation of a definition of only to show students how a formal definition addresses the idea of "closeness" and how it excludes concern for what occurs when x=a. (Optional)
11. Continuity and graphical properties of continuous functions (including Intermediate Value Theorem and Extreme Value Theorem).
12. Investigating functions that are not continuous at x = a.

The Derivative and Applications of Differentiation

Time: Approximately 45 days

Note: Students find little new in the definition of the derivative since they have been working with it from day one. Time is spent, however, in looking at what occurs when a function is not differentiable at a point (a, f(a)). We explain the non-differentiability both graphically and by noting reasons why the conditions required by the definition fail in particular cases.

Students will be required to analyze curves, including notions of monotonicity and concavity, optimization both relative and absolute, modeling rates of change, use of implicit differentiation with inverse functions, interpretation of the derivative as a rate of change including velocity, speed, and acceleration, and geometric interpretation of differential equations using slope fields.

Students will be able to compute derivatives of basic functions including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions. They will be able to use basic rules for computing derivatives of sums, products, and quotients of functions.

1. Definition of the Derivative.
2. Instantaneous rate of change (as the limiting value of average rate of change).
3. Investigating functions differentiable at x = a as well as those not differentiable at x = a algebraically, graphically, and using written communication.
4. Using f ' to investigate or confirm the increasing/decreasing behavior of f both algebraically, graphically, and using written communication.
5. The relationship between the graphs of f and f ' . Given one graph, sketching the other.
6. Relative (local) extrema and the first derivative test.
7. Absolute extrema in the context of applied problems.
8. Derivatives of Algebraic Functions
    - The Power rule is usually "discovered" in the introductory unit and now supported with proof.
    - The Sum, Constant Multiple, and Product rules are prompted by geometric arguments but are supported with proof.
9. Derivatives of circular functions.
10. Review of composite functions.
11. The Chain rule prompted by guided investigation and then support ed with proof.

12. Implicit Differentiation.
13. Related Rates.
14. The Second Derivative: concavity and points of inflection.
15. Making connections between f , f ', and f" in tables and in graphs.
16. Local linearity revisited.
    - Using linearization to approximate transcendental function values.
    - Using slope fields to better understand the significance of differential equations and, perhaps, to discover the general behavior of a function that is a solution to a differential equation by knowing an initial condition.
17. The relationship between differentiability and continuity.
18. The Mean Value Theorem (without proof).

The Integral and Applications of Integration

Time: Approximately 45 days

Note 1: Several days are spent in the first phase of this unit. Using graphing technology and available programs, we carefully explore Riemann sums. Although the investigation of area is the basic context, we include regions below the axis as well as those above. Students discover that Riemann sums are not always good predictors of the answer if area is the question. Then studying area, we discover that a Riemann sum is providing us with the sum of "signed" areas. The groundwork laid then carries over into applications involving distance as well, where a Riemann sum over an interval is able to provide displacement results, but not necessarily total distance traveled.

Note 2: Techniques of integration are de-emphasized. I want students to be able to find antiderivatives that are "reasonable," but we get most of our practice by dealing with applied problems rather than by doing pages of drill problems that I used to assign. It is important for students to occasionally encounter functions whose antiderivatives cannot be found, or cannot easily be found. This encourages them to use numerical techniques or to intelligently use the capabilities of available technology.

1. Riemann sums (left hand, right hand, and midpoint sums only) to approximate the area of a region bounded by continuous functions.
2. Evaluating limits of Riemann sums over equal subdivisions to determine area of regions bounded by polynomial functions on the interval [0,b].
3. The definite integral as a limit of Riemann sums.
4. The Fundamental Theorem of Calculus (with proof).
5. Indefinite integrals: antiderivatives of known functions and using simple substitutions.
6. Integration by parts.
7. Numerical approximations to definite integrals using tables and graphs.
    - review Riemann sums
    - Trapezoidal rule
8. Using definite integrals whose integrands are velocity functions to show that accumulating rates of change in distance yields net distance traveled.
9. Rectilinear motion

10. Volumes of known cross-sections as limits of Riemann sums, including
    - sums of discs
    - sums of washers
    - sums of cylindrical shells
    - sums of other cross-sectional slices.
11. Average value of a function.
12. Variable separable differential equations involving simple polynomial and trigonometric functions.

Transcendental Functions

Time: Approximately 30 days

1. Defining ln(x) =
2. Properties of natural logarithms
3. Logarithmic differentiation
4. Inverse functions and their derivatives
5. Exponential functions as Inverses of logarithmic functions
6. The definition of e
7. Differentiation and Integration involving e^u, a^u, and $\log_a u$.
8. Exponential growth and decay problems
9. Inverse Trigonometric functions and their derivatives
    - Review of all previously studied concepts and applications using transcendental functions.

Pedagogical Issues

I encourage my students to explore and discover as much as possible. Now I do much less lecturing than I did in the past. I find that students are very comfortable with graphing calculators, and that they are asking the questions that I used to have to ask because they are "seeing" many of the concepts with the help of our investigations and, of course, their calculators. They sit in groups of three, four, or five, and are encouraged to help each other by acting as mentors.

Homework assignments deal not only with current topics, but also with at least one review question as well. There is some time allowed at the beginning of class for them to try to help each other with questions from the previous night's assignment. This gets them talking about the mathematics and also cuts down on the number of questions with which we have to deal as a class. Students are responsible for getting help from me or from a classmate on any issues that we do not have time to address during class.

Three days before a test, I announce the cut-off of material for the next test so that students have several days to be sure that they have gotten all of their questions addressed. The tests are cumulative throughout the year, although the emphasis of the test is on the most current material. I use quizzes more as a means of helping students discover misconceptions early rather than as significant factors in their grade. I collect homework to be able to give each student feedback on the process and level of communication presented. This is only done on selected assignments, and is seldom done for a grade

Students who enroll in an AP level class are expected to take the AP Examination but are not required to do so. Some tests that I give make use of appropriate multiple choice questions taken from released AP Exams. Often, however, I will tell students that I will give partial credit if they show their work as well.

Graphing Calculators

As noted previously, my students consider their graphing calculators to be an integral tool for understanding the concepts of calculus. There are many programs available for graphing calculators that have been written by clever, insightful programmers, but I do not use many of them. Instead, I find that my students often develop their own programs. If I had a toolkit of pre-written programs for them, they might not have the motivation to explore on their own, nor would they have the ownership that comes with being the author, or knowing the author, of a helpful program. For example, in the past they have written programs to deal with algorithmic applications, such as Newton's method or the Trapezoidal rule. The more able programmers have also developed visual materials that actually draw rectangles used in finding Riemann sums. Thus, I have found that for most of the current syllabus topics, the TI-89 provides all the basic "needs" and students can supplement most of the programming "wants." As much as my students and I enjoy and profit from having this technology available, I am also not willing to give up too quickly on the pen-and-pencil approach to supplement our work. Although students often "see" concepts more easily with technological tools, they often need to think through the concepts and applications slowly, step-by-step, to really achieve mastery. I do not always know what questions to ask to best assess students' level of understanding of a concept or process when they have a calculator at their disposal, especially with the advent of calculators with symbolic capabilities such as the TI-89. Thus, there are times that we put the calculator aside and think about the graph or ponder the result as when we probe and share the joy of discovering that we put the symbolic capabilities of calculators aside, and instead share the joy of discovering that for $x>0$, $\ln(x) + C$. In summary, I am still getting my best "read" of what students know and what questions still remain for them, and I am still helping my students enjoy wonderful "aha!" moments by employing the best from both technological and non-technological realms.

During the course, students will primarily use the TI-89 calculator as a result. Every student will be provided a TI-89 calculator to use during the course. Students will learn how to use the calculator to compute derivatives and integrals as well as using it for analysis of graphs. Winplot from Peanut Software will be used for analyzing graphs, too. Its ability to graph implicit as well as explicit functions makes it a valuable tool.

Student Activities

I have developed an activity using the first hill of "The Beast" roller coaster at Kings Island near Cincinnati, Ohio. Given the function for the sinusoidal path of the hill, the students are required to compute extrema, inflection points, intervals of increasing and decreasing, intervals of concavity, and arc length. In other words, students are to thoroughly analyze the graph of the function on the given interval. The activity also emphasizes the importance of technology for computing integrals, since some functions may be too complex to compute by hand.

Successful completion of the activity involves written communication as students are required to explain and communicate understanding of the concepts in a paper. Each student is required to turn in a paper on the topic, and they are encouraged to expand on the problem by giving additional information. For instance, they may want to design a similar coaster of their own. To truly achieve a good grade on the assignment, the students must go beyond the prompt in some manner.

Additional activities including written communication will be assigned during the course as need dictates. If I determine that students are struggling with a particular topic, I generally make a written assignment. I have found that explain a process in writing leads the students to delve deeply into the concept assisting understanding and application of the content.